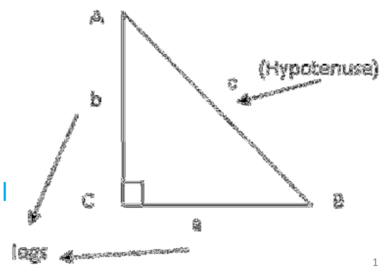
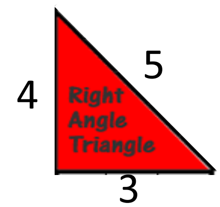
1.1 Pythagorean Theorem

- In a right triangle, the side opposite the 90° angle is called the **hypotenuse** and the remaining two sides are called the **legs**.

Note: Vertices are labeled with CAPITAL LETTERS while, sides are labeled with small letters.



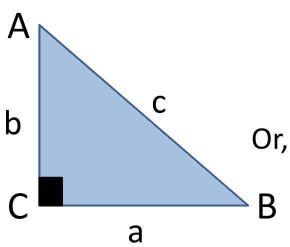
The great Greek Mathematician Pythagoras discovered an interesting relation between the side lengths of the right triangle.



Pythagorean Theorem: If triangle ABC is a right triangle, then

$$c^2 = a^2 + b^2$$

$$c = \sqrt{(a^2 + b^2)}$$

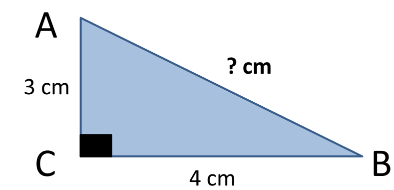


Or, if we are solving for the leg.

$$a^2 = c^2 - b^2$$

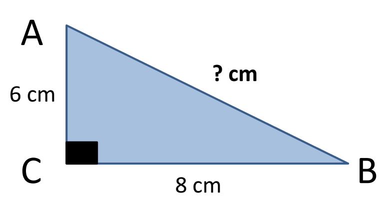
$$a = \sqrt{(c^2 - b^2)}$$

Ex 1: find the missing side length



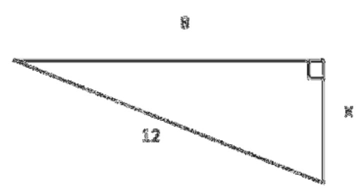
X here is a hypotenuse
So we use $c^2 = a^2 + b^2$

Ex 2: find the missing side length



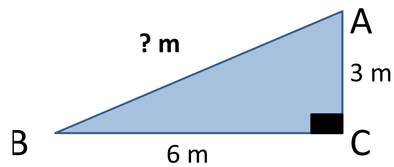
X here is a hypotenuse
So we use $c^2 = a^2 + b^2$

Ex 3: find the missing side length



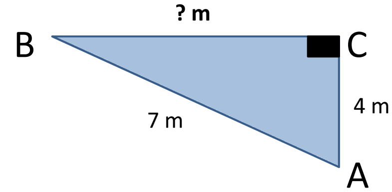
X here is a leg
So we use $a^2 = c^2 - b^2$

Ex 4: find the missing side length



X here is a hypotenuse
So we use $c^2 = a^2 + b^2$

Ex 5: find the missing side length



X here is a leg
So we use $a^2 = c^2 – b^2$

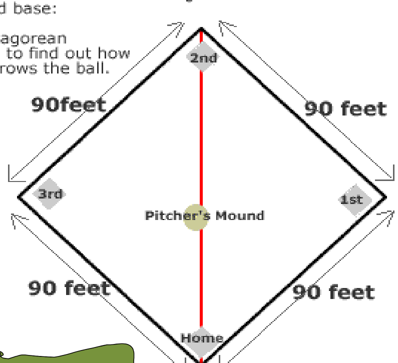
Practice: Page 4
1(a,d), 2(a,d), 3-8

The converse of the Pythagorean Theorem

- The reverse is also true that is if $c^2 = a^2 + b^2$ then triangle ABC is a right triangle with angle C = 90^0
- Some common Pythagorean triples are:
  {3,4,5} , {5,12,13} , {8,15,17} , {9,40,41}
- and their multiples like
  {6,8,10} , {9,12,15} and {10,24,26} etc.

A baseball diamond is really a 90 foot square.
Let's say the catcher is throwing the ball from home to second base:

Use Pythagorean Theorem to find out how far he throws the ball.



Practice: Page 6
9-12, 14, 15, 17

1.2 –A- Rational Numbers

There are different types of numbers:

- Real Numbers:
  - Natural
  - Integers
  - Rational
  - Irrational
- Complex Numbers (aka Imaginary Numbers)

Definitions:

$\mathbb{N}$ **:** Set of **Natural numbers** : {0,1,2,3,...}

$\mathbb{N}^*$ **:** Set of non zero natural numbers : {1,2,3,...}

$\mathbb{Z}$ **:** Set of **Integers** : {...,-3,-2,-1,0,1,2,3,...}

$\mathbb{Z}^*$ **:** Set of non zero Integers : {...,-3,-2,-1, 1,2,3,...}

$\mathbb{Z}_+$ **:** Set of positive Integers: {0,1,2,3,...) same as $\mathbb{N}$

$\mathbb{Z}_-$ **:** Set of negative Integers: {...,-3,-2,-1,0}

$\mathbb{Q}$ **:** Set of **Rational numbers**

(i.e. numbers that can be written as fractions including terminating decimals (0.5 = $\frac{1}{2}$), and repeating decimals ($0.\overline{5}$ = $\frac{5}{9}$)

Definitions:

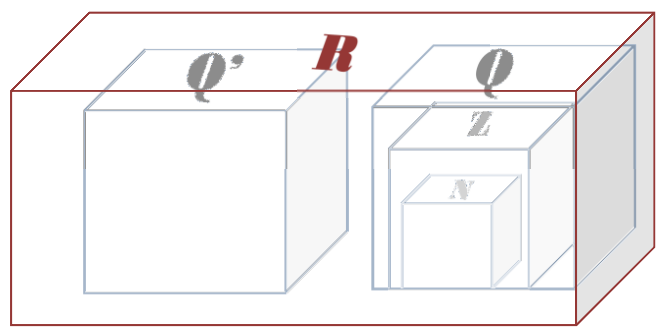
$\mathbb{Q}'$ **:** are Irrational Numbers, these are non-periodic (non-repeating), non-terminating decimals; so we cannot write them as fractions.

(Ex: π , $\sqrt{2}$, $\sqrt{3}$, $\sqrt[3]{4}$ etc.)

$\mathbb{R}$ **:** is the Set of Real Numbers , that is all Rational and Irrational numbers: **Q** ∪ **Q'**

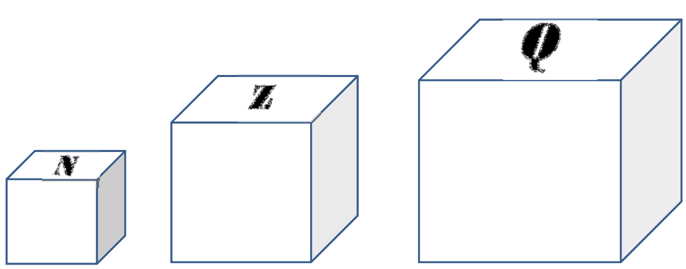
We read this: **Q** Union **Q** prime.

So if we were to put them in nesting boxes (or circles) they would look like this:



Ex 1 : place each number in the correct box.

0 0.3 -3 -7 5 -2/3 100



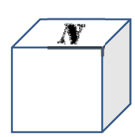
⊆ means subset; ϵ means element of

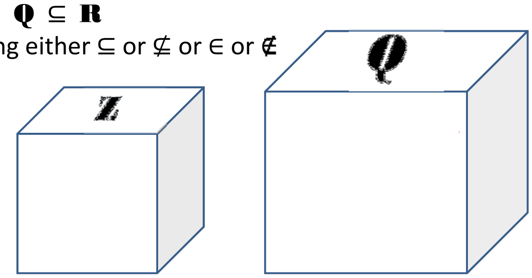
$\mathbb{N}$ is a subset of $\mathbb{Z}$ is a subset of $\mathbb{Q}$ is a subset of $\mathbb{R}$

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

Ex 2: fill using either ⊆ or ⊈ or ∈ or ∉

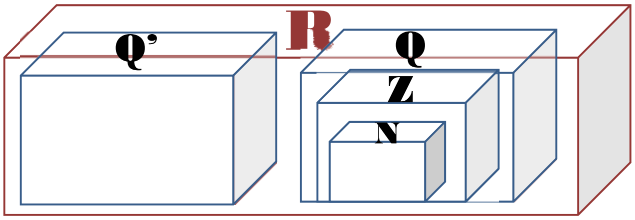
$\mathbb{Q}$ ___ $\mathbb{Z}$



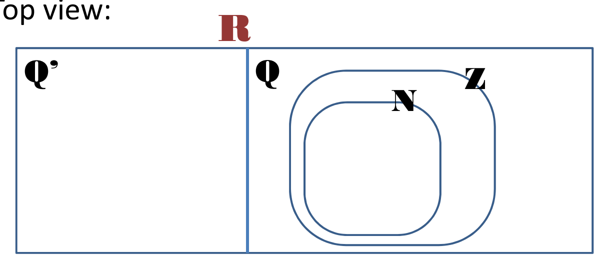


-3 ___ $\mathbb{Z}$

−3 ___ $\mathbb{N}$

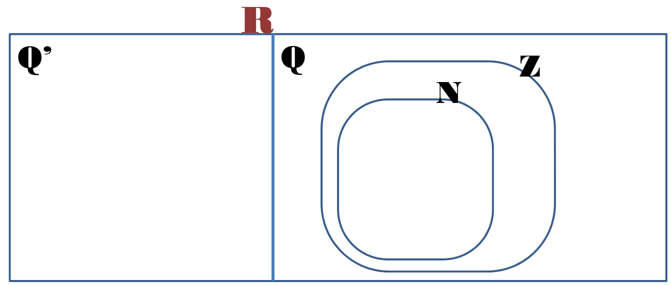


Top view:



Ex 3 : place each number in the correct box.

-1 $-0.\overline{6}$ $-\sqrt{5}$ 11/7 -12 $\sqrt{4}$ 0.5 π 10 $\sqrt{2}$



Practice:
page 10 # 1-3
page 22 # 1-3

1.2 –B- Writing a Rational number as Fractions or Decimals

Case 1: From Fraction to Decimal

- Use a calculator
- The **period** of a rational number is the infinitely repeating decimal(s) --we put a bar over it.

Ex 1:

a) $\frac{16}{11}$ = the period is

b) $\frac{63}{55}$ = the period is

c) $\frac{1}{2}$ = the period is

Case 2: From Decimal to Fraction

Ex 2: Write the following terminating decimals as reduced fractions.

a) 0.3 =

d) 2.05 =

b) 1.22 =

e) 5.0025 =

c) 0.225 =

f) 3.012 =

Trick: $\frac{x}{9} = 0.\overline{x}$ and $\frac{xy}{99} = 0.\overline{xy}$

Ex 3: Write the following repeating decimals as reduced fractions.

a) $0.\overline{3}$ =

d) $2.\overline{05}$ =

b) $1.\overline{23}$ =

e) $5.\overline{0025}$ =

c) $0.\overline{225}$ =

f) $3.\overline{012}$ =

Practice:
page 11 # 4(a-e)
page 13 # 5(a,b,c), 6(a,d)

If period is not right after decimal point!!

Ex 3: write $0.1\overline{6}$ as a reduced fraction

Method 1:

$$0.1\overline{6} \xrightarrow{\times 10} 1.\overline{6} = 1\frac{6}{9} = 1\frac{2}{3} = \frac{5}{3} \xrightarrow{\div 10} \frac{5}{30} = \frac{1}{6}$$

Reduce and change to improper fraction

We multiply by 10 to get the period alone after the decimal point.
Since that changes the value we have to undo it later by dividing by 10 again.
Dividing by 10 means just add the zero in the denominator.

Method 2: to write $0.1\overline{6}$ as a reduced fraction

Explanation:	Steps:
1) Make an equation –write the period twice	1) Let $x = 0.16\overline{6}$
2) Multiply both sides by a power of 10 to move decimal point to after 1 period	2) $100x = 16.\overline{6}$
3) and again to before one period.	3) $-\ 10x = 1.\overline{6}$
4) Subtract the 2 equations	4) $90x = 15.0$
5) Solve for x ; and reduce the fraction if possible	5) $x = \frac{15}{90}$ $= \frac{1}{6}$

Ex 4: write $1.2\overline{36}$ as a reduced fraction

Method 1:

Method 2:

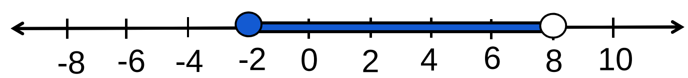
Practice:
page 13 # 5(d), 6(b,c), 8

1.5 Intervals

Brackets are very important in math and they mean different things. There are 3 types

Round:	P(0,5)	an ordered pair (x=0,y=5)	order is important so, it is not the same as (5,0)
Curly:	S= {0,5}	a set of 2 elements That is 0 & 5 ∈ S	order is not important so, it is the same as {5,0}
Square:	I= [0,5]	an Interval. That is all the real numbers from 0 to 5	same as saying $0 \le x \le 5$

Number Line



● Filled: the end number IS in the set

○ Not filled/Empty: the end number IS NOT in the set

▬ Identifies the interval of numbers

Interval Notation (square brackets)

[-2, 8 [

Lowest # ↗ ↖ Highest #

[-5, 7] 

FACING/HUGGING brackets mean the end number is CONTAINED in the set.

] 5, 9 [

BACK FACING brackets means the end number is NOT CONTAINED in the set.

Set builder Notation {Inequalities}

Review: fill in the correct sign so that x is

Less than 5 __________

Greater than 10 __________

At Most 22 __________

At Least 15 __________

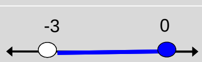
We read the inequality from left to right.

EFF RULE:
Equal sign
Filled circle
Facing bracket

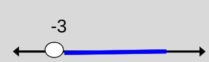
$\{x \in R \mid -2 \le x < 8\}$

x is a real # — Lowest # — Highest #

Bounded Intervals

Interval	Set-Builder	Number Line
[0,3]		
	$\{x \in R \mid -1 \le x < 5\}$	
		
	$\{x \in R \mid a < x < b\}$	

Unbounded Intervals

Interval	Set-Builder	Number Line
]-∞,3]		
	$\{x \in R \mid x \ge 3\}$	
		
	$\{x \in R \mid x < b\}$	

Practice Page 26 # 1-5

1.6 Natural Number Exponents
-A- Power of a Real Number

Expanded form : $2\times2\times2\times2\times2\times2\times2 = 128$

<u>Exponential form</u>:

Exponent

Base → $2^7 = 128$ ← Power

∴ $a^n = a\times a\times\ldots\times a$ (n times)

Note that:

$a^1 = a$ $a^0 = 1$

$a^n =$ **<u>if</u>** the 'a' is negative and 'n' is odd

Ex: $(-2)^3 = (-2)(-2)(-2) = -8$

$(-a)^n = (-a)\times(-a)\times\ldots\times(-a)$ (n times)

$-a^n = -a\times a\times\ldots\times a$ (n times)

Ex 1: Expand and evaluate:

a) $7^3 =$

b) $x^4 =$

c) $9^1 =$

d) $5^0 =$

e) $(-8)^4 =$

f) $-8^4 =$

g) $(7)(6)(7)(6) =$

Ex 2: Evaluate:

a) $2 \times 10^3 + 6 \times 10^2 + 7 \times 10 + 3$

b) $\dfrac{2^4 + 5^2}{3^3}$

Ex 3: Folding Paper Experiment

# Folds	# Sections	Expanded Form	Exponent Form
1	2	2	2^1
2	4	2x2	
3			
4			
5			
6			
7			
n			

<u>Claim</u>: A dry piece of paper cannot be folded in perpendicular halves more than seven times, regardless of its size

Mythbusters http://www.youtube.com/watch?v=kRAEBbotuIE

Practice:
Page 28 # 1 - 8,
9(abc), 10(abc), 11(abc)

1.6-B- Laws of Exponents

Question	Expanded Form	Exponent	Law
$5^2 \times 5^4$			$a^m \times a^n =$
$2^5 \div 2^2$			$a^m \div a^n =$ If m>n
$2^2 \div 2^5$			$a^m \div a^n =$ If m<n
$(4^3)^2$			$(a^m)^n =$
$(5a)^4$			$(ab)^n =$
$\left(\frac{3}{4}\right)^2$			$\left(\frac{a}{b}\right)^n =$

Why is $a^0 = 1$

$$5^3 \div 5^3 = \frac{5 \times 5 \times 5}{5 \times 5 \times 5} = \frac{125}{125} = 1$$

$$5^3 \div 5^3 = 5^{3-3} = 5^0$$

Therefore $5^0 = 1$

Ex 1: Simplify these exponents (DO NOT EVALUATE)

$9^7 \times 9^6 =$ ______

$3^7 \times 3 =$ ______

$4^7 \times 4^{-3} =$ ______

$7^0 \times 7^{12} =$ ______

$2^{22} \times 2^{-20} =$ ______

$(-6)^7 \times (-6)^6 =$ ______

$3^7 \div 3 =$ ______

$9^7 \div 9^6 =$ ______

$4^7 \div 4^{-3} =$ ______

$\frac{7^{12}}{7^9} =$ ______

$2^{22} \div 2^{-20} =$ ______

$\frac{(-6)^5}{(-6)^7} =$ ______

Ex 2: Simplify these exponents

$(2^7)^2 =$ ______

$(3^3)^2 =$ ______

$(a^4)^3 =$ ______

$(q^n)^m =$ ______

$(5^5)^5 =$ ______

$((-3)^2)^4 =$ ______

Ex 3: Simplify these exponents

$\frac{(x^3)^2}{x^2} =$ ______

______

$\frac{(2^3)(a^2)^3(b^3)^3}{4a^6b^8}$ ______

______

Ex 4: Express each power as a new power with the given base

$16^2 =$ ______ $= 2^{\_\_}$

$16^2 =$ ______ $= 4^{\_\_}$

$25^3 =$ ______ $= 5^{\_\_}$

$27^3 =$ ______ $= 3^{\_\_}$

Ex 4: Solve $4^4 \div 2^2 =$

Ex 5: Simplify these exponents

$\frac{3a^2}{9a^4}$ ______

$\frac{15a^5}{25a^2}$ ______

$\frac{3a^4}{6b^2} \times \frac{2a^2}{b^4} \times \frac{b^9}{a^5}$ ______

Ex 6: Simplify

$(3a)^6 =$ ______

$(6b)^3 =$ ______

$(2a^3)^2 =$ ______

$(a \times b)^n =$ ______

$\left(\frac{5}{3}\right)^4 =$ ______

$\left(\frac{7}{x}\right)^3 =$ ______

$\left(\frac{b}{8}\right)^2 =$ ______

$\left(\frac{a}{b}\right)^n =$ ______

Ex 4: Fill in with = or ≠

1) $5^2 + 5^3$ $\quad 5^5$
2) $2^4 \times 2^3$ $\quad 2^{12}$
3) $4^2 \times 5^3$ $\quad 20^5$
4) $5^2 + 5^2$ $\quad 10^4$
5) $5^6 - 5^2$ $\quad 5^4$
6) $6^4 / 2^2$ $\quad 3^2$
7) $5^6 / 5^2$ $\quad 5^3$
8) x $\quad x^0$

Ex 7: Simplify

12 f) $(5x^3)^2 =$ ______

13 f) $(3a^4b^2)^2\ (-a^5b)^2\ (2a^3b^2)^2$

$=$ ______

$=$ ______

15d) $\left(\frac{3x^2}{y^3}\right)^3 \left(\frac{y^4}{9x^4}\right)^2 =$ ______

Ex 8: Extra Practice - Simplify

a) $\frac{(3x^3y^5)^2}{3x^2y^4}$

$=$ ______

b) $\frac{(4a^2b^3)^3}{(2a^3b^4)^2}$

______

Evaluate at a = -2 and b = -1

c) $\frac{(5a^6b^4)^3}{(5a^2)^3}$

$=$ ______

d) $\frac{(6x^2y^3)^3}{(2x^4y^2)^2}$

______

Practice:
Page 30-35 # 1– 10, 12, 13, 15 – 17
only (acegik) of each question

1.7 Negative Exponents

Warm up: Evaluate using the calculator

2^2 =

2^{-2} =

-2^2 =

-2^{-2} =

$(-2)^2$ =

$(-2)^{-2}$ =

$(2^{-2})^2$ =

$\left(\frac{1}{2}\right)^{-2}$ =

Let's see what a negative exponent means…

$$4^3 \div 4^5 = 4^{3-5} = 4^{-2}$$

$$4^3 \div 4^5 = \frac{4\times4\times4}{4\times4\times4\times4\times4} = \frac{1}{4^2}$$

What I call
FLIP AND SWITCH
FLIP the base, and SWITCH the exponent to positive

$$a^{-n} = \frac{1}{a^n}$$

A base raised to a NEGATIVE exponent is equivalent to 1 over the original base with the same exponent but positive .

EX 1: Write with a positive exponent

$a)\ 3^{-2} =$ ______

$b)\ 2^{-2} =$ ______

$c)\ \left(10^2\right)^{-2} =$ ______

$d)\ p^{-3} =$ ______

$e)\ \frac{a^3}{a^7} =$ ______

$f)\ \frac{b^5}{b^{10}} =$ ______

More examples page 36

2(e) $\left(\frac{3a^{-2}}{b^4}\right)^{-2}$

5(f) $\left(\frac{3^3 3^{-2}}{3^{-1}}\right)^{-2}$

8(b) 0.01

More examples page 36

9(i) $(0.01)^2(0.1)^{-1}$

10(e) $(a^2b^{-1})(a^{-2}b^3)$

11(e) $\left(\frac{a^4}{2b^{-2}}\right)^{-3}\left(\frac{4b^{-1}}{a^8}\right)^{-2}$

Practice:
Page 36+ # 1 - 13

1.8 Scientific Notation

Warm up: Perform the following operations

1) 14 ÷ 10 =

2) 482 ÷ 1000 =

3) 662 ÷ 10^2 =

4) 127 x 10 =

5) 48023 x 10^4 =

1.8 Scientific Notation

When numbers get really Big or really small it is inconvenient to write out all of the zeros.

For this reason we use *Scientific Notation*.

A positive number in **scientific notation** is in the form:

$a \times 10^n$ where $1 \le a < 10$; and n is an integer.

Ex 1: Express in scientific notation $a \times 10^n$ $1 \le a < 10$

POSITIVE EXPONENT

a) 5600

NEGATIVE EXPONENT

b) 0.00042

Ex 2: Write the following numbers in scientific notation:

1) 360 =

2) 0.4 =

3) 7523 =

4) 45000 =

5) 235 =

6) 1226000 =

7) 0.025 =

8) 0.000045 =

9) 81 =

Examples 1:

42 000 000 000 000 x 72 000 000 000

Examples 2:

42 000 000 000 x 0.000 000 000 21

Examples 3:

125 000 000 000 ÷ 0.000 000 000 25

Examples 4:

42 000 000 000 ÷ 0.000 000 000 126

Practice:
Page 40 # 4-10

1.9 Rational Exponents

Warm up:

1) $7^2 =$

2) $\sqrt{49} =$

3) $3^3 =$

4) $\sqrt[3]{27} =$

5) $5^4 =$

6) $\sqrt[4]{625} =$

7) $(-3)^3 =$

8) $\sqrt[3]{-27} =$

9) $\sqrt[3]{8} =$

10) $\sqrt[4]{81} =$

11) $\sqrt[4]{-16} =$

12) $\sqrt[3]{-125} =$

If $b^n = a$	then $\sqrt[n]{a} = b$		
	or $a^{\frac{1}{n}} = \sqrt[n]{a} = b$		

Note that $\sqrt[n]{a}$ does not exist if n is even and a < 0

Example: $\sqrt[2]{-64}$ and $\sqrt[4]{-64}$ don't exist

Where as $\sqrt[3]{-64} = -4$, because
$(-4)^3 = (-4)(-4)(-4) = -64$

Ex 1: Evaluate:

1) $169^{\frac{1}{2}} =$

2) $(-169)^{\frac{1}{2}} =$

3) $-169^{\frac{1}{2}} =$

4) $125^{\frac{1}{3}} =$

5) $(-125)^{\frac{1}{3}} =$

Ex 1: Evaluate:

6) $-125^{\frac{1}{3}} =$

7) $\sqrt[3]{2^3} =$

8) $144^{-\frac{1}{2}} =$

9) $\sqrt[3]{-\frac{8}{27}} =$

Ex 2: Simplify:

a) $\sqrt[3]{\frac{8x^9}{x^3}}$

b) $\left(\frac{9a^{-2}}{b^4}\right)^{-\frac{1}{2}}$

$=$

$=$

$=$

Practice:
page 42 # 1 - 6